which, using [2], becomes

$$\rho_m \frac{\partial u_m}{\partial t} + \rho_m u_m \frac{\partial u_m}{\partial x} + \frac{\partial}{\partial x}\left[\frac{\rho_1 \rho_2}{\rho_m}(u_1 - u_2)^2\right] = -\frac{\partial P}{\partial x}. \quad [30]$$

Thus one concludes that a term similar to Soo's inertial interaction term should appear in the mixture momentum equation and not in the separate phase equations. Applying [30] to the vapor–droplet flow field discussed above leads to

$$\frac{\mathrm{d}}{\mathrm{d}x}(\rho_2 u_2^2) \approx -\frac{\mathrm{d}P}{\mathrm{d}x} \quad [31]$$

which states that the loss of droplet momentum dur to aerodynamic drag is balanced by a pressure gradient.

The "inertial interaction" term in [30] is directly equivalent to the "apparent stress" arising from diffusion encountered in the momentum equations for a mixture of gaseous species (Penner 1957; Truesdell & Toupin 1960; Woods 1975). In the latter the "inertial interaction" term is frequently combined with the shear-stress tensor (Penner 1957). This practice can lead to significant difficulties in the case of disperse two-phase flows.

Acknowledgements—The author acknowledges helpful discussions with Messrs. W. Comfort and T. Alger regarding these equations.

REFERENCES

Basset, A. B. 1888 *A Treatise on Hydrodynamics*, Vol. 1. Deighton, Bell & Co. (republished by Dover, New York, 1961).

Bird, R. B. *et al.* 1960 *Transport Phenomena*. John Wiley, New York.

Crowe, C. T. 1977 On the vapor–droplet flows including boundary-droplet effects. *Proc. Two-Phase Flow and Heat Transfer Symposium-Workshop* (National Science Foundation, University of Miami, 1976).

Drew, D. A. 1971 Average field equations for two-phase media. *Studies in Applied Mathematics*, Vol. L, No. 2. Massachusetts Institute of Technology, Cambridge, MA.

Penner, S. 1957 *Chemistry Problems in Jet Propulsion*. Pergamon Press, New York.

Soo, S. L. 1976 On one-dimensional motion of a single component in two phases. *Int. J. Multiphase Flow* **3**, 79–82.

Truesdell, C. & Toupin, R. 1960 The classical field theories. *Encyclopedia of Physics* (Edited by Flügge S.). Springer, Berlin.

Woods, L. C. 1975 *The Thermodynamics of Fluid Systems*. Clarendon, London.

Int. J. Multiphase Flow, Vol. 4, pp. 229–231. Pergamon/Elsevier 1978. Printed in Great Britain

REPLY TO PROFESSOR CROWE: "ON SOO'S EQUATIONS, etc."

S. L. Soo

Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, U.S.A.

I am honored by Professor Crowe's attention to my work. It seems that his proving a fundamental relation via a specific example is contrary to the deductive nature of mechanics. However, some insight is actually gained when his example is solved correctly. Professor Crowe's finding appears to be a result of his liberal use of "$\approx$'s".

Professor Crowe's present example could have many interesting conditions depending on the heat and mass transfer relations. To be correct, his "absorbing wall" should not arbitrarily remove momentum. If the liquid is partially evaporated at the "absorbing wall" we would have, at that wall:

$$\rho_1 u_1 + \rho_2 u_2 = \rho_2' u_2'$$

where $\rho_2' u_2'$ is the mass flow into the wall. This will make his u_m non-zero and is not what he wanted.

Professor Crowe has chosen the case where all the liquid evaporates as it reaches the "absorbing wall" and the vapor flows backward thus achieving a steady flow system. We shall see what happens when his [12] and [13] are substituted into the momentum equation of phases without his approximations. The momentum equation of phase 2 after combining with its continuity equation gives:

$$\frac{\partial \rho_2 u_2^2}{\partial x} - \Gamma_2 u_2 = -\frac{\partial P\phi_2}{\partial x} + I_{21} + V_{21} \qquad [15S]$$

if ϕ_2 is treated as a variable.† Here and subsequently [15S] means [15] in Professor Crowe's paper which is rewritten after corrections.

The exact form of I_{21} is now

$$\begin{aligned} I_{21} &= \frac{\partial}{\partial x}\left[\frac{\rho_1^2 \rho_2}{(\rho_1+\rho_2)^2}(u_1-u_2)^2\right] - \frac{\rho_1}{\rho_1+\rho_2}(u_2-u_1)\Gamma_2 \\ &= \frac{\partial}{\partial x}(\rho_2 u_2^2) - u_2\Gamma_2. \end{aligned} \qquad [17S]$$

The "≃" in his [17] is unnecessary. The correct form of his [20] should be:

$$0 = \frac{\partial P\phi_2}{\partial x} + V_{21} \qquad [20S]$$

exactly. Similarly, for phase 1, we get

$$0 = -\frac{\partial P\phi_1}{\partial x} + V_{12}. \qquad [21S]$$

The sum of [20S] and [21S] gives

$$0 = \frac{\partial P}{\partial x} \qquad [22S]$$

since Professor Crowe's [12] gives $u_m = 0$; [22S] satisfies [8], as it should. His [22] is a consequence of "≈" in his [15]–[19], and is hence a wrong conclusion. A physical reason against his approximation is that the two phases are linked by his [12] and the presence of neither can be neglected.

This brings up the question one might raise on the meaning of his [30]. It gives, for $u_m = 0$,

$$\frac{\partial}{\partial x}(\rho_2 u_2^2 + \rho_1 u_1^2) = -\frac{\partial P}{\partial x} \qquad [30S]$$

†Soo, S. L. 1977 Multiphase mechanics of single component two phase flow. *Physics Fluids* **20**, 568–570.

exactly instead of his [31]. Note that [30S] gives a pressure drop without a meaningful cause or even a meaningful direction of the gradient. Professor Crowe, therefore, has managed to prove that his [30] is not a correct relation. His physical interpretation of his [31] appears to be a conjecture. For a thorough discussion of inertial coupling in multiphase mechanics, readers are referred to the paper by Chao *et al.* in this issue of the Journal.

Int. J. Multiphase Flow, Vol. 4, p. 231. Pergamon/Elsevier, 1978. Printed in Great Britain

AUTHOR'S CLOSURE

The change in the form of pressure gradient term in the momentum equation [3] in Professor Soo's original paper, (Soo 1976) and his "corrected" version in the comments [15S] is noteworthy.

CLAYTON T. CROWE